

# Interest Rate Modeling With Random Regimes

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## INTEREST RATES

## STRESS TESTING

## MODELS

## 20 Year U.S. Treasury Rates



# ANECDOTAL HISTORY

## 1974: Guaranteed Income Contracts in the Group Pension Market



# ANECDOTAL HISTORY

Early 1978: 12% Will Bring Blood In the Streets



# ANECDOTAL HISTORY

February 1980: It Didn't; October 1981: Even 15.13% Didn't



### FAILURE TO "SEE" A PLAUSIBILITY WAS CATASTROPHIC FOR INSURERS

- One Of The Largest Was Bankrupted
  - An Acquisition Disguised The Fact
- At Least One Of The Largest Visited The Fed
  - ... For A "Maybe, What If?" Discussion
  - Turned Out To Be Unnecessary

## Saving Grace: Double-Digit Rates Only Lasted For 6 Years





## SET TODAY'S STAGE

- Dynamic Valuation Interest Rates
- Asset Adequacy Analysis
- Appointed Valuation Actuary
- Principles-Based Discussion Began

# ANECDOTAL HISTORY

## January 1993: The Fed Won't Tolerate Long Rates Below 4%



# ANECDOTAL HISTORY

January 1999: Maybe They Will, But That's What RBC Is For



# ANECDOTAL HISTORY

January 2003: No, That's What Asset Adequacy Reserves Are For



# FACTUAL HISTORY ~ FUTURE QUESTION

December 2008: How Long Under 4%?; June 2011: Measured From When?



- STRESS-TESTING CAN APPLY TO
  - Reserves
    - Asset Adequacy Testing
    - Risk Management for Product/Line of Business
  - Surplus
    - Risk Based Capital
    - Embedded Value
  - Economic Capital
    - Basel II (III, etc.)
    - Own Solvency and Risk
  - Enterprise Risk Management

## STRESS-TESTING IS NOT

- A BY-PRODUCT OF FORECASTING
  - Forecasting Looks For Most Likely Outcomes
    - Maybe Within A Confidence Band
  - Forecasting Supports Current Decision-Making
  - Forecasting Will Be Judged By Actual Accuracy

## STRESS-TESTING IS NOT

- A BY-PRODUCT OF PRICING
  - Pricing Looks For Expected Values
    - Usually With Reasonable Variance Bounds
  - Pricing Supports Product Portfolio Development
  - Pricing Will Be Judged By Average Accuracy



## STRESS-TESTING IS

- A SEPARATE, DISTINCT DISCIPLINE
  - One That Looks For Extreme Values
    - Beyond Reasonable Variance or Confidence
    - But Within The Realm Of Plausibility ( ??? )
    - ....(any fool can assume that the sky will fall)
  - One That Supports Institutional Resilience
  - One That Will Be Judged By "No Surprises"

## STRESS-TESTING REQUIRES

- Truly Severe Values
  - Threats To Survival
    - Firm Not Providing Value If These Are Not "In Sight"?
    - ...(Or Maybe We're Not "Seeing" Very Well?)
  - On Both Extremes
- That Are Somehow Still Plausible
  - By What Standard?
    - History?: at a minimum
    - Theory?: maybe
    - Judgment?: be very wary of setting a maximum
  - Informed By History, Theory, and Judgment

## STRESS-TESTING **CAN** / **SHOULD** IGNORE

- Accuracy
  - Around Likely Or Expected Scenarios
- Current Wisdom & Judgment
  - About Variance And Confidence Bands
- The Arbitrage-Free Shibboleth
  - If Someone Couldn't Get Rich Is It Truly Extreme?
  - But Do Preserve Both Extremes
- Risk-Neutral Modeling
  - Risk-Neutral Models Predict Today's Prices
  - Risk-Neutral Distributions Are Make-Believe

## Deterministic Interest Scenarios

- Necessary But Maybe Not Sufficient
- Risk That It's Limited By Current Imagination
  - "There Would Be Blood In The Streets"
- Risk That It's Limited By Historical Extremes
  - But If It Already Happened Isn't Worse Plausible?
- How Do We Know How Bad Is Bad Enough?
  - Yet Still Plausible

# Back To The Anecdotes - 1978

12% Exceeded The Bounds Of Both History And Imagination



# Back To The Anecdotes - 1978 To 1980/81

## Imagination and History Were Not Nearly Enough

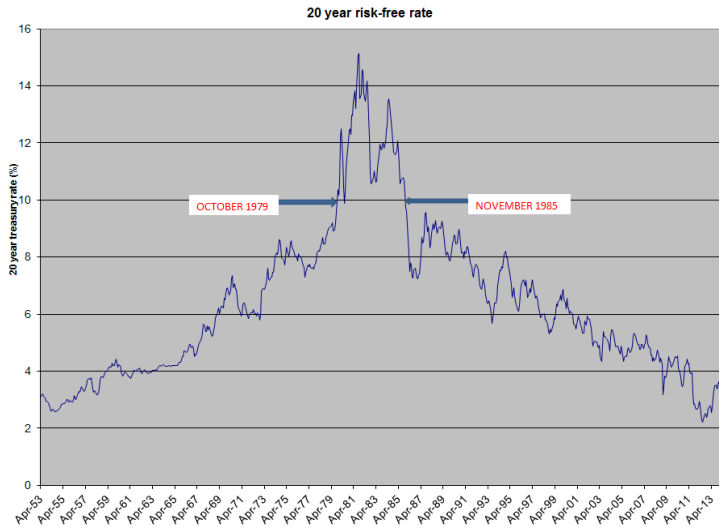


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- How Do We Know How Bad Is Bad Enough?
  - Yet Still Plausible
  - OR HOW LONG IS LONG ENOUGH? (Plausibly)

# FACTUAL HISTORY

Remember: Double-Digit Rates Only Lasted For 6 Years





# Back To The Anecdotes - 1990's

Japan Told Us That  $< 2\%$  or  $3\%$  Deterministic Was Plausible



# Back To The Anecdotes - Flash-Forward

So 2008 & 12 Were Not A "Surprise" For The Deterministic Stress-Test



### BUT FOR HOW LONG $< 2\%$ or $3\%$ ?

- Forever?: Not Plausible  
(Remember 6 Years Of Double-Digit Rates?)
- 5 - 10 Ten Years?: Maybe Not Severe Enough?
- We Finally Resorted To Random Scenarios
- Definitely A Last Resort
  - We'd Seen Too Much Abuse of Stochastic Models
  - They Only Give Back What You Put In
    - But Hard To Recognize Own Input Coming Back At You
  - No-Arbitrage and Risk-Neutral All The Rage
    - Risked Confusing Even Knowledgeable Audience

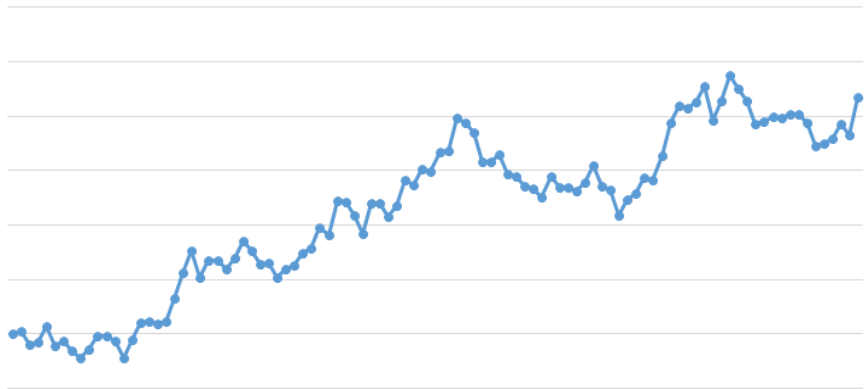
## RANDOM INTEREST RATE SCENARIOS

- The Extreme Scenarios Will Be The Stress-Test
  - So Risk-Neutral And Arbitrage-Free Are Irrelevant
- Start With A Model For An Anchor Rate
  - 20 Year Treasury
  - Build A Yield Curve Off That Later
- Choices
  - Pure Dispersion (Random Walk)

# The Choices Pictorially

PURE DISPERSION – RANDOM WALK — IMPLAUSIBLE

$$\Delta \ln Rate_t = \text{Gaussian} \Delta$$



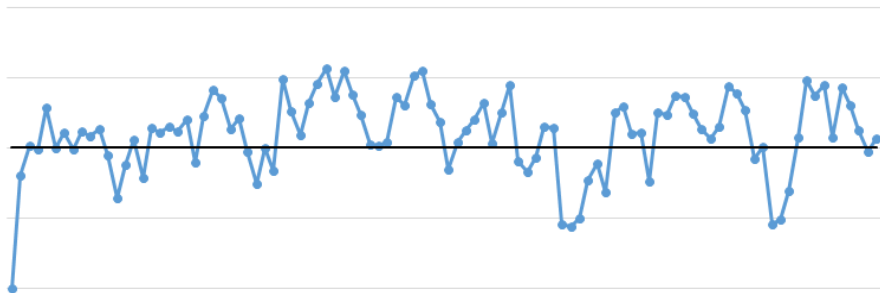
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  - Pure Dispersion (Random Walk) ... Implausible
  - Introduce A Mean Reversion Point (MRP)

# The Choices Pictorially

INTRODUCE A MEAN REVERSION POINT (MRP)

$$\Delta \ln Rate_t = F * (MRP - \ln Rate_{t-1}) + (1 - F) * Gaussian\Delta$$



# RANDOM INTEREST RATE SCENARIOS

- The Extreme Scenarios Will Be The Stress-Test
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- Choices
  - Pure Dispersion (Random Walk) ... Implausible
  - Introduce A Mean Reversion Point (MRP)
    - Which One (MRP) and How Fast (F)?
    - Any Choices Eliminate Some Historical Extremes –
    - (Either Level Extremes &/Or "How Long?" Extremes)
    - AAA Generator Chose This (And Eliminated Both)



## RANDOM INTEREST RATE SCENARIOS

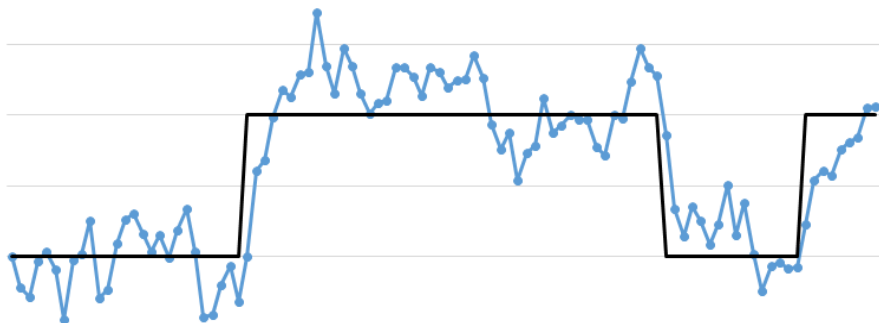
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  - Introduce More Than One MRP (Regimes)
    - Switch Off Among Them (Somehow Randomly)

# The Choices Pictorially

INTRODUCE MORE THAN ONE MRP (REGIMES)

$$\Delta \ln Rate_t = F * (MRP_i - \ln Rate_{t-1}) + (1 - F) * Gaussian\Delta$$

$i = 1,2$  deterministic when regime switch randomly occurs



## RANDOM INTEREST RATE SCENARIOS

- More Choices
  - Introduce More Than One MRP (Regimes)
    - Switch Off Among Them (Somehow Randomly)
    - How Many? At What Levels? With What Frequency?
    - Assumptions & Output Both Look Artificial
    - Little Or No Guidance From Interest Rate History
    - ... (How much worse than 15% / 2% is plausible?)

## RANDOM INTEREST RATE SCENARIOS

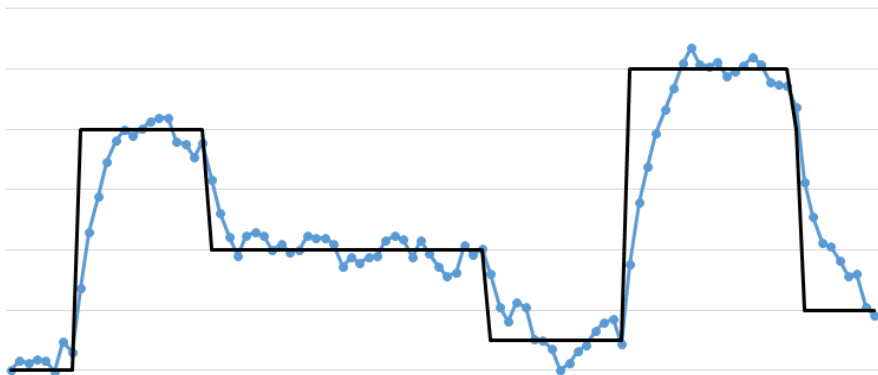
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  - Introduce A Random Field Of MRPs (Regimes)
    - Switch Off Among Them (Somehow Randomly)

# The Choices Pictorially

INTRODUCE A RANDOM FIELD OF MRP<sub>s</sub> (REGIMES)

$$\Delta \ln Rate_t = F * (MRP_t - \ln Rate_{t-1}) + (1 - F) * Gaussian\Delta$$

$MRP_t$  random when regime switch randomly occurs



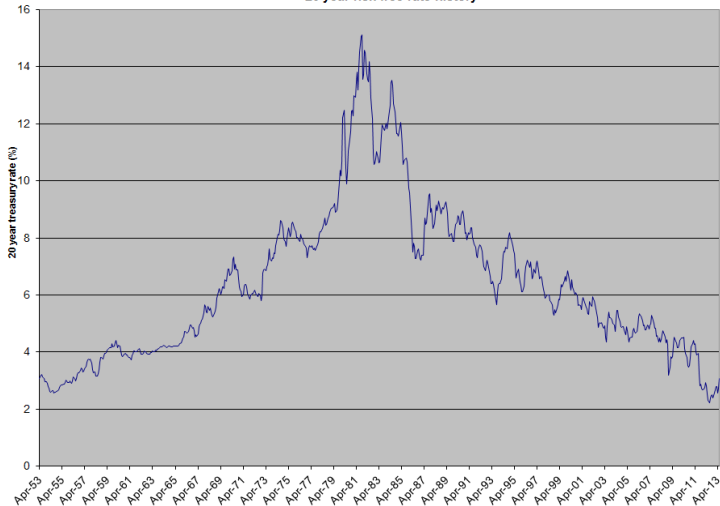
# RANDOM INTEREST RATE SCENARIOS

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    - ... (How much worse than 15% / 2% is plausible?)
  - Introduce A Random Field Of MRPs (Regimes)
    - Switch Off Among Them (Somehow Randomly)
    - Output Starts To Look Very Natural / Assumptions(??)
    - Historical Extremes Fit Right In
    - We Chose This One – Parameters A Challenge

# History of 20 Year US Treasury Rate

Plausible By Definition

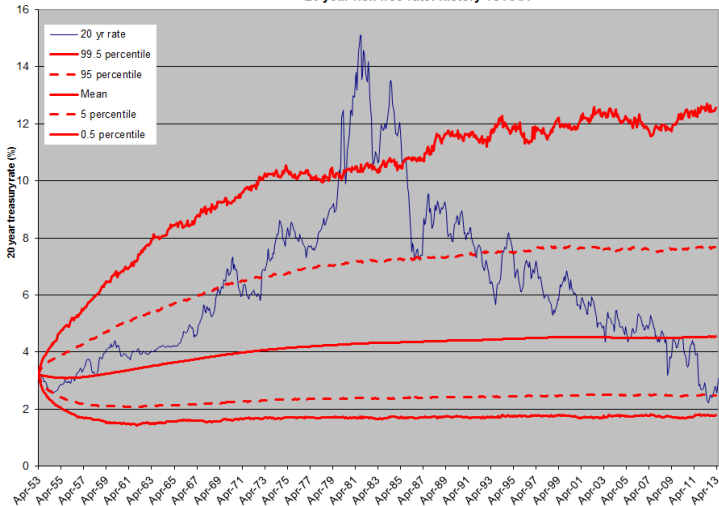
20 year risk-free rate history



# M'ly %-iles Apr.2013 AAA Generator (NAIC MRP 4.25%)

Neither Early 80's Nor Japan Are Remotely Plausible In AAA

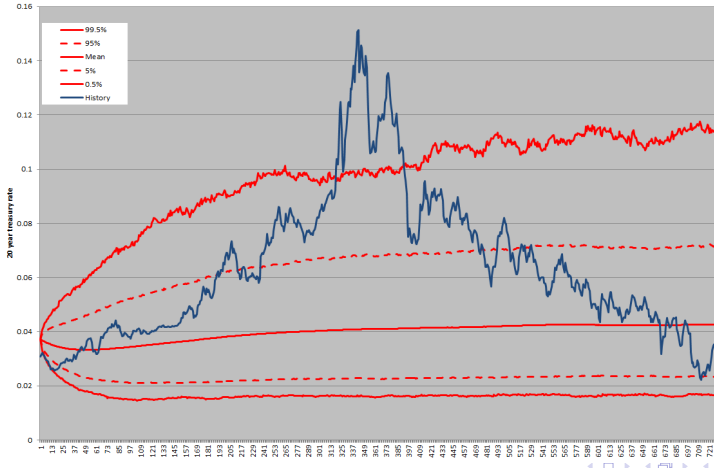
20 year risk-free rate: history vs AAA





## Update To New MRP Makes It Worse Even With Higher Starting Rate

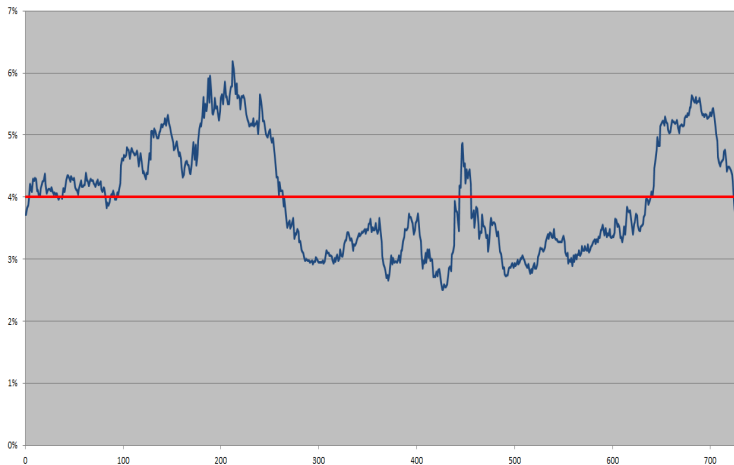
20 yr risk free rate history April 1953 to February 2014 vs AAA starting January 2014



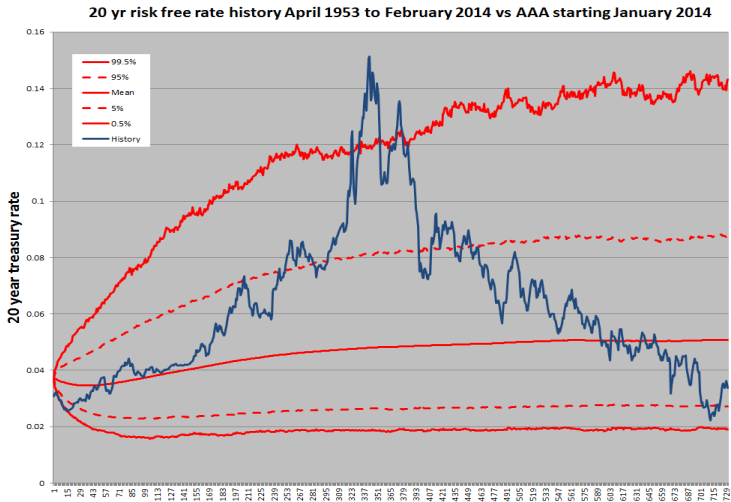
# 99%-ile Scenario: Dec.2013 AAA (NAIC MRP 4.00%)

No One Scenario Hugs The Bottom – Here's 99% Cumulative < 4% Run

99th Percentile Worst Scenario: No. 9913 - Fixed MRP Eliminates Long Runs < 3% By Its Very Design

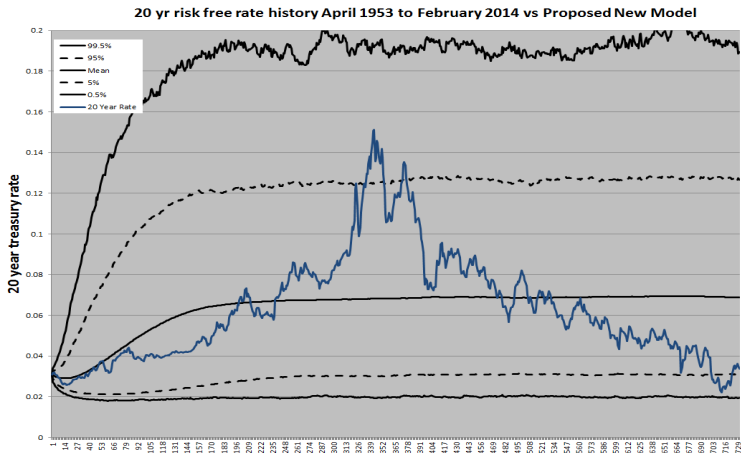


## AAA's Recommended MRP Helps A Little, But Loses At The Bottom



# M'y %-iles Randomized MRPs / Apr. 1953 Start

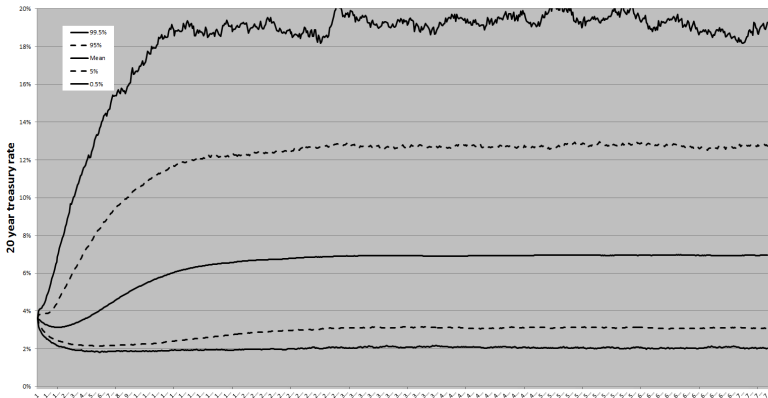
Extreme Enough To Envelop History – But Still Plausible



# M'y %-iles Randomized MRPs / Dec. 2013 Start

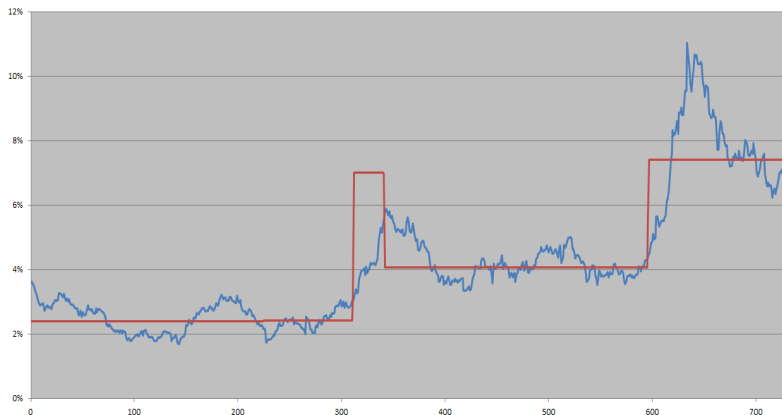
Starts Higher But Still Has Similar Range Of Plausibilities

Proposed New Model Dec. 2013 - 60 Years



## And Model Design Has Not Automatically Ruled Out Bottom-Hugging

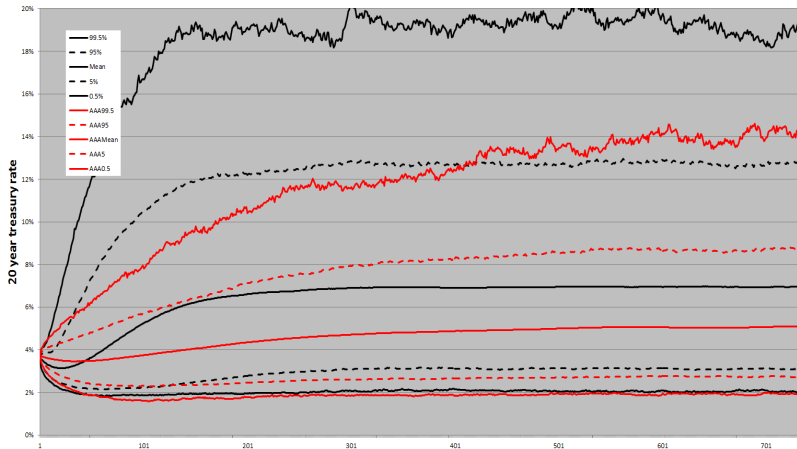
99th Percentile Worst Scenario: No. 866 - Random MRP Opens Up Long Runs < 3%



# Randomized MRP vs AAA (with AAA 4.75% MRP)

## The High-Rate Risk Is Captured Much Better

Randomized MRP vs AAA 4.75% MRP Dec. 2013 - 60 Years



FROM 1994 TO 2006:

— A LOT OF TRIAL & ERROR

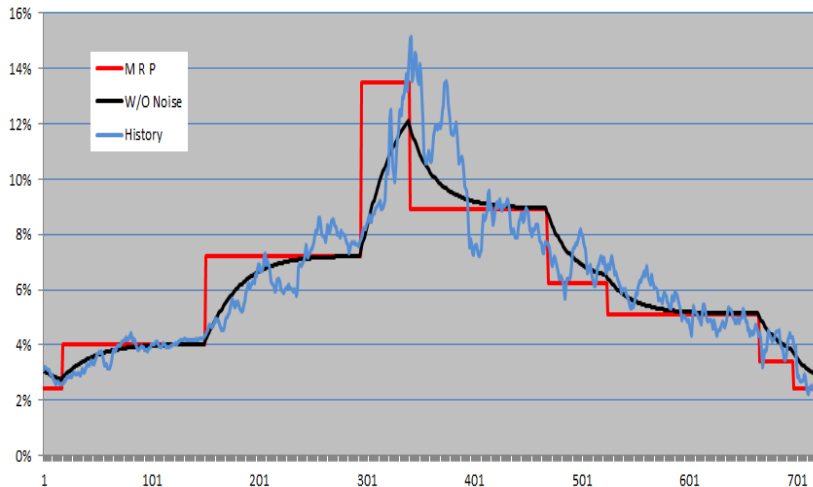
SINCE THEN:

— SOME ATTEMPT AT SCIENCE



# Calibration Of The Randomized MRP Model

## Historical Regimes – Derived From A Filtering Procedure



## WAITING TIME TO REGIME SWITCH

- The MLE Gamma Distribution On Historical
  - Only 8 Data Points
    - $\text{Alpha} = 3.52$ ;  $\text{Beta} = 2.32$
    - $\text{Mean} = \text{Alpha times Beta} = 8.2 \text{ Years}$
  - Alpha And Beta Low Confidence Separately
    - Mean Is Really What Affects Model Output Anyway
  - Interesting That  $\text{Mean} = \text{US Political Cycle}$

## DISTRIBUTION OF MRP

- Assume Lognormal - Mutually Independent

## REVERSION SPEED

- Set Jointly With The Lognormal Parameters
  - To Get Best Fit With Moments Of Historical Rates

# Calibration Of The Randomized MRP Model

## Rate Levels and Spread Align With History

	<b>61 Year History</b>	<b>Model Mean</b>	<b>Model StdDev</b>
<b>Rate = 20 Year Treasury</b>			
Rate Mean	.0631	.0638	.0131
Rate StdDev	.0266	.0266	.0109
Rate Kurtosis (normal=3)	3.54	2.92	1.24
Rate 6th-osis (normal=15) (6th Ctrl Mom/StdDev <sup>6</sup> )	21.7	15.5	19.3

## VOLATILITY OF INTEREST RATES

- History Is Not Clearly Lognormal
  - We Fit A Three Parameter DiGeneralized Gamma
  - Using L1 & L2 Distances Of Cumulative Distributions
- Essentially As Good As AAA Generator
  - Which Fits Historical Volatility Very Well Indeed
  - Using Stochastic Volatility & Yield Curve Dynamics

- Many Generations of Actuarial Students
  - At Aetna Life Insurance Company
  - At AnTai Life Insurance Company (in Taiwan)
  - At Aetna International Inc.
- Many Generations of UConn Students
  - Master's In Mathematics, conc. Actuarial Science
  - Master's In Applied Financial Mathematics
- Most Recently
  - Songchen (Darren) Zhang
  - Zepeng (Ben) Xie
  - Xuezhi (Kevin) Zhang
  - Nyan Paing Tin